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Erdős–Rényi Models and Neutrosophic Sets

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Abstract

Networks comprise a vast array of diverse entities, including organizations, computers, airports, and more. The elements are interconnected either physically (e.g., through cables or microwaves) or mentally, when they pertain to ideas or concepts. In the broadest context, the linkages exhibit no discernible pattern. Graphs are mathematical constructs utilized to represent networks and analyze their characteristics. A random graph is one in which a stochastic process dictates the existence of an edge between two vertices. An Erdős–Rényi model is a framework for generating random graphs or examining the evolution of a random graph. Substituting *likelihood* with *plausibility* results in a fundamentally distinct framework that can be mathematically characterized using fuzzy graphs. In addition, we gain far more flexibility by adding a measure of the opposite of plausibility and a measure of *indeterminacy*. This naturally leads to a reinterpretation of established concepts and ideas and the framework of neutrosophic set theory.

Keywords: Erdős–Rényi Model, Neutrosophic Graphs, Random Graphs

Introduction

*N*etworks are omnipresent and comprised of interconnected nodes. A network doesn't have to be a physical thing; it might be just any relation between objects. For instance, various airliners connect airports in a network. The reader should consult [1] for a clear discussion of several intriguing features of networks.

Graphs are abstract networks in which only the number of nodes and the connections between them are relevant. The theory of graphs is well-developed, and there are several books to master graph theory (see, e.g., [2]). It is feasible to build a graph by means of a random process that, given a set of nodes, determines if there exists a connection between two arbitrary nodes [3]. These types of graphs are referred to as random graphs, and their theory is significant; various methods exist for constructing them. The most prominent of these models is the Erdős–Rényi model [4, 5, 6].

Fuzzy sets are a mathematical description of vagueness. A property is vague if we can't agree on its presence. The idea of fuzzy sets is based on the assumption that a human, a plant, an animal, or, more generally, an object has some property to some degree. In most circumstances, this degree is a number that belongs to the set 0,1. In fuzzy mathematics, when an item possesses a property to a degree equal to d, then the same object does not have this property to a degree equal to $1 - d$. Let's change this criterion to require that the two degrees be independent but that their sum be between 0 and 1. Let us denote by $T_{A(x)}$ the membership degree of x and by $F_{A(x)}$ the non-membership degree of x. Then, the degree of nondeterminacy is defined as $I_{A(x)} = 1 - T_{A(x)} - F_A(x)$. We can, however, assume that this degree does not depend on the other two degrees, and the only restriction is that $0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3$. The resulting structures are called

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neutrosophic sets in literature [7]. These structures are an extension of "intuitionistic" fuzzy sets [8].

A Neutrosophic Graph is a graph in which vertices and/or edges are represented by neutrosophic sets [9, 10]. This allows modeling not only the presence or absence of connections but also the degree of uncertainty and indeterminacy in those connections. Such graphs are particularly useful in situations where information is incomplete, inconsistent, or vague—conditions often encountered in real-world decision-making systems and social networks [10]. By incorporating the indeterminacy aspect, neutrosophic graphs provide a richer and more flexible framework than fuzzy or intuitionistic fuzzy graphs for representing complex systems.

Formally, a neutrosophic graph $G = (V, E)$ is defined such that each vertex $v \in V$ and/or edge $e \in E$ is associated with a triplet T, I, F, where $T, I, F: V \to [0,1]$ and

$$0 \leq T(v) + I(v) + F(v).$$

The incorporation of indeterminacy makes neutrosophic graphs a powerful extension of fuzzy models, opening up new avenues for graph-theoretic research and its applications in uncertain environments.

In this paper we present random neutrosophic graphs that are a natural extension of random "intuitionistic" graphs [11].

Gentle Introduction to Neutrosophic Graphs

In many real-world systems, relationships among entities are often vague, incomplete, or even contradictory. While classical Graph Theory offers a robust framework to model networks, and fuzzy graphs extend this framework to handle uncertainty using membership values between 0 and 1, they still assume that uncertainty can be represented using a single degree of membership. This assumption may not hold in highly complex environments where truth, indeterminacy, and falsity may simultaneously coexist to varying degrees.

As was noted above, Florentin Smarandache introduced neutrosophic set theory to address this limitation. Formally, a neutrosophic set A on universe X is defined as:

$$A = \{(x, T_A(x), I_A(x), F_A(x) \mid x \in X\}.$$

A *neutrosophic graph* is an octuple

$$G = (V, E, T_V, I_V, F_V, T_E, I_E, F_E),$$

where each vertex $v \in V$ and each edge $(u, v) \in E$ has an associated neutrosophic triple. To maintain consistency, we impose:

$$T_E(u, v) \leq \left(T_V(u), T_V(v)\right),$$
$$F_E(u, v) \geq \left(F_V(u), F_V(v)\right),$$
$$0 \leq I_E(u, v) \leq 1.$$

Definition 1. A neutrosophic graph G is an ordered pair V, E where V is a nonempty set of vertices and $E \subseteq V \times V$ is a set of edges. Each vertex $v_i \in V$ is associated with a neutrosophic membership triple $(T_{v_i}, I_{v_i}, F_{v_i})$, and each edge $e_{ij} \in E$ is associated with a triple $\left(T_{e_{ij}}, I_{e_{ij}}, F_{e_{ij}}\right)$ satisfying:

$$T_{e_{ij}} \leq \left(T_{v_i}, T_{v_j}\right), I_{e_{ij}} \geq \left(I_{v_i}, I_{v_j}\right), F_{e_{ij}} \geq \left(F_{v_i}, F_{v_j}\right)$$

for all $v_i, v_j \in V$.

Remark 1. If $T_{v_i} = 1$ and $I_{v_i} = F_{v_i} = 0$ for all $v_i \in V$ and $T_{e_{ij}} = 1$, $I_{e_{ij}} = F_{e_{ij}} = 0$ for all $e_{ij} \in E$, the neutrosophic graph reduces to an ordinary crisp graph. If only T and F values are used with $I = 0$, the neutrosophic graph reduces to an intuitionistic fuzzy graph.

Example 1. Suppose $V = \{a, b, c\}$ with vertex neutrosophic memberships

$$T_a = 0.8,$$
$$I_a = 0.1,$$
$$F_a = 0.1,$$
$$T_b = 0.6,$$
$$I_b = 0.3,$$
$$F_b = 0.1,$$
$$T_c = 0.7,$$
$$I_c = 0.2,$$
$$F_c = 0.1.$$

Assume edges $E = \{(a,b),(b,c)\}$ with neutrosophic edge memberships

$$(T_{ab}, I_{ab}, F_{ab}) = (0.6,0.2,0.2),$$
$$(T_{bc}, I_{bc}, F_{bc}) = (0.5,0.3,0.2).$$

Then $G = (V,E)$ forms a neutrosophic graph.

Definition 2. A neutrosophic subgraph $G' = (V', E')$ of G is such that $V' \subseteq V$ and $E' \subseteq E$. A path in a neutrosophic graph is a sequence of distinct vertices $v_p, v_{i_1}, \dots, v_{i_k}, v_q$ such that $(v+p, v_{i_1}), (v_{i_1}, v_{i_2}), \dots, (v_{i_k}, v_q) \in E$. If the path begins and ends at the same vertex, it forms a cycle. A path or cycle is simple if it does not contain the same edge more than once.

Neutrosophic graphs allow the simultaneous representation of known, unknown, and contradictory information, which is often encountered in decision-making, social networks, and biological systems. They have shown superiority over classical, fuzzy, and intuitionistic fuzzy graphs in modeling real-world systems where indeterminacy plays a crucial role [3, 4, 9]. In classical graphs, an edge is either present or absent between any two vertices. However, in many real-world situations, the relationship between vertices is not crisp but involves degrees of truth, falsehood, and indeterminacy. To handle such uncertainty, Neutrosophic Set theory, introduced by Florentin Smarandache [7], provides a powerful mathematical framework. This theory generalizes the concept of fuzzy set and intuitionistic fuzzy set by associating three membership degrees—truth (T), indeterminacy (I), and falsity (F)—with each element.

Applying this concept to graphs leads to the definition of a neutrosophic graph. Here, each vertex and/or edge is assigned a triplet of values representing its degree of truth, indeterminacy, and falsity.

Definition 3. A single-valued neutrosophic graph (SVNG) G is a triple V, E, σ where:

- V is a nonempty finite set of vertices.
- $E \subseteq V \times V$ is the set of edges.
- σ is a function that assigns to each vertex $v \in V$ a triplet $\sigma(v) = (T_v, I_v, F_v)$ and to each edge $e \in E$ a triplet $\sigma(e) = (T_e, I_e, F_e)$, where $T_e, I_e, F_e \in [0,1]$ and $0 \le T_e + I_e + F_e \le 3$.

Remark 2. Neutrosophic graphs can model situations where there is incomplete, inconsistent, and indeterminate information about the presence of a relation. This makes them particularly suitable in domains such as decision-making, social network analysis, medical diagnosis, and semantic web modeling [12]. Note that SVNGs are based on single-valued neutrosophic theory.

Example 2. Let $V = \{v_1, v_2, v_3\}$ be a set of vertices and $E = \{\{v_1, v_2\}, \{v_2, v_3\}\}$ be the set of edges. Assign neutrosophic values as follows:

$$\sigma(v_1) = (0.7,0.1,0.2),$$
$$\sigma(v_2) = (0.6,0.3,0.1),$$
$$\sigma(v_3) = (0.8,0.1,0.1),$$
$$\sigma(\{v_1, v_2\}) = (0.5,0.4,0.2),$$
$$\sigma(\{v_2, v_3\}) = (0.7,0.2,0.1).$$

Then, $G = (V, E, \sigma)$ is a simple neutrosophic graph.

Definition 4. A neutrosophic subgraph $G'$of a neutrosophic graph G is a graph such that:

- $V(G') \subseteq V(G)$;
- $E(G') \subseteq E(G)$; and
- For all $x \in V(G') \cup E(G')$: $\sigma_{G'}(x) \le \sigma_G(x)$ component-wise.

Neutrosophic Erdős–Rényi models

Networks appear widely in physical, biological, social, and conceptual systems. As networks consist of interconnected nodes, their relationships may not follow a deterministic pattern. Classical graph theory [13] abstracts networks using vertices and edges, while random graphs, introduced through the Erdős–Rényi models [11], describe probabilistic edge formation.

Fuzzy sets represent vagueness through membership degrees, and fuzzy entity–relationship models extend probabilistic graphs by incorporating fuzzy memberships to represent uncertainty in relationships between entities. However, fuzzy sets cannot fully represent indeterminacy or inconsistency. Neutrosophic Set Theory introduces three independent components—truth (T), indeterminacy (I), and falsity (F)—each ranging independently in 0,1. This enables modeling of incomplete, contradictory, and ambiguous network information.

The $NER - G(n, M)$ model

Given a neutrosophic graph G:

- Select n vertices.
- Choose exactly M edges uniformly.
- Assign neutrosophic values (T_0, I_0, F_0) to selected edges.
- For all unselected edges: $(T_E, I_E, F_E) = (0,1,1)$.

This generalizes the fuzzy $G(n, M)$ model where unused edges were assigned membership 0 and non-membership 1.

The $NER - G(n, p)$ Model

Instead of probability p, potential edge is assigned:

$$(T_E, I_E, F_E) = \left(T_p, I_p, F_p\right),$$

representing the degrees to which the edge exists, is indeterminate, or does not exist.

Conclusion

This paper introduced Neutrosophic Erdős–Rényi Graph Models by extending fuzzy Erdős–Rényi graph models. By assigning neutrosophic triples to vertices and edges, the models can capture complex uncertainty, incomplete information, and contradictory relationships. Future research may explore connectivity probabilities, phase transitions, degree distributions, and algorithmic properties of neutrosophic random graphs.

Ethics approval

Not required

Competing interests

All the authors declare that there are no conflicts of interest.

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Underlying data

Derived data supporting the findings of this study are available from the corresponding author on request.

Declaration of artificial intelligence use

We hereby confirm that no artificial intelligence (AI) tools or methodologies were utilized at any stage of this study, including during data collection, analysis, visualization, or manuscript preparation. All work presented in this study was conducted manually by the authors without the assistance of AI-based tools or systems.

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