Journal of Analytical Uncertainty
Vol. 1, No. 1, December 2025 Page. 28 - 34
E-ISSN: xxxx-xxxx | P-ISSN: xxxx-xxxx
https://doi.org/10.63924/

A Variable-Indeterminacy Weighted Correlation Framework in Neutrosophic Statistics

Prasanta Kumar Raut

Department of Mathematics, Trident Academy of Technology, Bhubaneswar, Odisha, India
*Corresponding author's email: prasantaraut95@gmail.com

Abstract

Correlation analysis under uncertain and imperfect information remains a fundamental challenge in statistical modeling. Although neutrosophic statistics provides a flexible framework for handling truth, falsity, and indeterminacy simultaneously, most existing neutrosophic correlation measures treat the indeterminacy component as uniform across observations. Such an assumption may oversimplify real data, where uncertainty often varies due to measurement errors, subjective assessments, or incomplete knowledge. In this paper, a weighted neutrosophic correlation framework with observation-dependent indeterminacy is developed. The proposed model allows each data point to possess its own indeterminacy level and incorporates weighting parameters to reflect the relative reliability or importance of observations. Based on these considerations, a new weighted neutrosophic correlation coefficient is formulated using neutrosophic means, variances, and covariance. Fundamental statistical properties of the proposed coefficient, including boundedness, symmetry, and invariance under linear transformations, are rigorously established. A detailed numerical illustration is provided to demonstrate the computational procedure and to show how variable indeterminacy and weighting influence the strength of association between variables. The proposed approach offers a more realistic and adaptable tool for analyzing relationships in uncertain environments and may serve as a useful foundation for further developments in neutrosophic data analysis and decision-making applications.

Keywords: Neutrosophic set, Variable indeterminacy, Weighted correlation, Uncertainty modeling

Article history: Submitted 12/10/2025|Revised 07/11/2025|Accepted 25/12/2025|Online first 28/12/2025

1. Introduction

Correlation analysis is one of the most widely used tools in statistics for measuring the degree of association between variables. Classical correlation measures, such as Pearson's coefficient, are effective when data are precise and fully reliable. However, in many realworld applications, observations are affected by vagueness, incompleteness, and subjective judgment, which limits the applicability of classical statistical methods. This limitation has motivated the development of mathematical frameworks capable of modeling uncertainty more realistically.

Fuzzy set theory was introduced to address imprecision in data, leading to early studies on correlation in fuzzy environments. Yu [1] investigated correlation measures for fuzzy numbers, while Chiang and Lin [2] proposed correlation concepts for fuzzy sets. These approaches extended classical correlation to situations involving gradual membership but did not explicitly account for hesitation or indeterminacy. To overcome this shortcoming, Atanassov [3] introduced intuitionistic fuzzy sets by incorporating both membership and nonmembership degrees. Correlation measures for intuitionistic fuzzy sets were subsequently developed in probability spaces by Hong and Hwang [4] and further analyzed by Hung and Wu [5], enhancing the descriptive capability of fuzzy correlation models.

Despite these advancements, fuzzy and intuitionistic fuzzy frameworks remain limited in representing indeterminate information explicitly. To address this issue, Smarandache introduced neutrosophic logic and neutrosophic sets [6, 7], which characterize uncertainty using three independent components: truth, indeterminacy, and falsity. This tri-component structure provides a more flexible and comprehensive mathematical model for uncertain and inconsistent information. The formal definition and generalization of neutrosophic sets were further elaborated in [8], establishing a strong theoretical foundation for subsequent developments. Salama and Al-Blowi [9] expanded this framework by introducing neutrosophic topological spaces, thereby enriching the structural properties of neutrosophic mathematics.

Building upon these foundations, neutrosophic statistics has emerged as an effective extension of classical and fuzzy statistical methodologies. Several correlation measures for neutrosophic data have been proposed to generalize traditional correlation concepts while incorporating indeterminacy. Hanafy et al. [10, 11] developed correlation coefficients for neutrosophic sets and investigated their behavior in probability spaces. The relationship between neutrosophic correlation and regression analysis was later explored in [12], demonstrating the applicability of neutrosophic correlation in statistical modeling. More recently, statistical correlation coefficients for single-valued neutrosophic sets have been proposed and successfully applied to medical diagnosis problems [13]. Related similarity and correlation-based measures for rough neutrosophic sets were discussed in [14], highlighting their usefulness in decision-making contexts.

Although existing neutrosophic correlation models effectively incorporate indeterminacy, most of them assume that the indeterminacy component remains constant across all observations. In practical situations, however, the degree of indeterminacy may vary from one observation to another due to measurement errors, incomplete information, or varying levels of reliability. Ignoring this variability may lead to loss of information and reduced analytical accuracy. Moreover, real data often require assigning different importance levels to observations, which is not adequately addressed in many existing models.

Motivated by these considerations, the present paper proposes a variable-indeterminacy weighted correlation framework within neutrosophic statistics. The proposed approach allows indeterminacy to vary across observations and incorporates weight parameters to reflect the relative importance or reliability of data points. By extending the classical notion of correlation to a weighted neutrosophic setting with observation-dependent indeterminacy, the developed framework provides a more flexible and realistic tool for analyzing associations under uncertainty. The proposed correlation coefficient preserves fundamental statistical properties while enhancing applicability to complex data analysis and decision-making problems involving indeterminate information.

2. Preliminaries

2.1. Neutrosophic Set

Let X be a universe of discourse. A neutrosophic set A in X is defined as

$$A = \{(x, T_A(x), I_A(x), F_A(x)) : x \in X\},$$

where $T_A(x)$, $I_A(x)$, and $F_A(x)$ represent the truth-membership, indeterminacy-membership, and falsity-membership degrees, respectively, satisfying

$$0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3.$$

2.2. Neutrosophic Variables with Variable Indeterminacy

Let $X = \{x_1, x_2, ..., x_n\}$ be a set of real-valued observations. A neutrosophic variable with variable indeterminacy is defined as

$$X_N = \{x_i + I_i : i = 1, 2, ..., n\},$$

where $I_i \in [0,1]$ denotes the indeterminacy associated with the i-th observation.

Similarly, another neutrosophic variable can be defined as

$$Y_N = \{y_i - I_i : i = 1, 2, ..., n\}.$$

3. Weighted Neutrosophic Correlation Coefficient

Let $X_N = \{x_i + I_i\}$ and $Y_N = \{y_i - I_i\}$ be two neutrosophic variables with variable indeterminacy. Let $w_i > 0$ be the weight associated with the i-th observation such that

$$\sum_{i=1}^{n} w_i = 1.$$

3.1. Definition

The weighted neutrosophic means are defined as

$$\bar{X}_N = \sum_{i=1}^{n} w_i (x_i + I_i), \qquad \bar{Y}_N = \sum_{i=1}^{n} w_i (y_i - I_i).$$

The weighted neutrosophic covariance is given by

$$\mathrm{Cov}_N(X, Y) = \sum_{i=1}^{n} w_i \left[(x_i + I_i - \bar{X}_N)(y_i - I_i - \bar{Y}_N) \right].$$

The weighted neutrosophic variances are

$$\sigma^2_{X_N} = \sum_{i=1}^{n} w_i (x_i + I_i - \bar{X}_N)^2,$$

$$\sum_{i=1}^{n} \quad \sigma^2_{Y_N} = \sum_{i=1}^{n} w_i (y_i - I_i - \bar{Y}_N)^2.$$

The weighted neutrosophic correlation coefficient is defined as

$$r_{WN} = \frac{\mathrm{Cov}_N(X,Y)}{\sqrt{\sigma^2_{X_N}\,\sigma^2_{Y_N}}}.$$

4. Properties of the Proposed Correlation Coefficient

4.1. Boundedness

The weighted neutrosophic correlation coefficient satisfies

$$-1 \le r_{WN} \le 1.$$

4.2. Symmetry

For any two neutrosophic variables X_N and Y_N,

$$r_{WN}(X_N, Y_N) = r_{WN}(Y_N, X_N).$$

4.3. Invariance under Linear Transformation

Let a,b,c,d be real constants with $a \neq 0$ and $c \neq 0$. Then

$$r_{WN}(aX_N + b,\ cY_N + d) = \frac{ac}{|ac|}\, r_{WN}(X_N, Y_N).$$

5. Numerical Illustration

In this section, a comprehensive numerical example is presented to illustrate the computation of the proposed weighted neutrosophic correlation coefficient with variable indeterminacy. Consider two sets of observations

$$X = \{12,9,8,10,11,13,7\}, \qquad Y = \{14,8,6,9,11,12,3\}.$$

Let the corresponding indeterminacy values associated with each observation be

$$I = \{0.2,0.1,0.3,0.2,0.1,0.2,0.4\}.$$

Assume equal importance of observations, and hence the weight vector is taken as

$$w_i = \frac{1}{7}, \qquad i = 1,2,\ldots,7.$$

Step 1: Construction of Neutrosophic Variables

The neutrosophic variables are defined as

$$X_N = X + I, \qquad Y_N = Y - I.$$

Thus, the transformed neutrosophic data are given in Table 1.

Table 1: Neutrosophic transformation with variable indeterminacy

i	x_i	y_i	$x_i + I_i$	$y_i - I_i$
1	12	14	12.2	13.8
2	9	8	9.1	7.9
3	8	6	8.3	5.7
4	10	9	10.2	8.8
5	11	11	11.1	10.9
6	13	12	13.2	11.8
7	7	3	7.4	2.6

Step 2: Computation of Weighted Neutrosophic Means

The weighted neutrosophic means are computed as

$$\bar{X}_N = \sum_{i=1}^{7} w_i(x_i + I_i) = \frac{1}{7}(12.2 + 9.1 + 8.3 + 10.2 + 11.1 + 13.2 + 7.4) = 10.79,$$

$$\bar{Y}_N = \sum_{i=1}^{7} w_i(y_i - I_i) = \frac{1}{7}(13.8 + 7.9 + 5.7 + 8.8 + 10.9 + 11.8 + 2.6) = 8.79.$$

Step 3: Computation of Weighted Neutrosophic Covariance

The weighted neutrosophic covariance is defined as

$$\mathrm{Cov}_N(X,Y) = \sum_{i=1}^{7} w_i \left\langle (x_i + I_i - \bar{X}_N)(y_i - I_i - \bar{Y}_N) \right\rangle.$$

The intermediate calculations are shown in Table 2.

Hence,

$$\mathrm{Cov}_N(X,Y) = \frac{1}{7}(7.06 + 1.50 + 7.69 - 0.01 + 0.65 + 7.25 + 21.00) = 6.73.$$

Table 2: Deviation and covariance computation

i	$x_i + I_i - \bar{X}_N$	$y_i - I_i - \bar{Y}_N$	Product
1	1.41	5.01	7.06
2	-1.69	-0.89	1.50
3	-2.49	-3.09	7.69
4	-0.59	0.01	-0.01
5	0.31	2.11	0.65
6	2.41	3.01	7.25
7	-3.39	-6.19	21.00

Step 4: Computation of Weighted Neutrosophic Variances

The weighted neutrosophic variances are calculated as

$$\sigma^2_{X_N} = \sum_{i=1}^{7} w_i(x_i + I_i - \bar{X}_N)^2 = 3.77,$$

$i=1$

$$\sigma_{Y_N}^2 = \sum_{i=1}^{7} w_i (y_i - I_i - \bar{Y}_N)^2 = 10.44.$$

Step 5: Computation of Weighted Neutrosophic Correlation

Finally, the weighted neutrosophic correlation coefficient is obtained as

$$r^{WN} = \frac{\text{Cov}_N(X,Y)}{\sqrt{\sigma_{X_N}^2 \, \sigma_{Y_N}^2}} = \frac{6.73}{\sqrt{3.77 \times 10.44}} = 0.93.$$

Interpretation

The obtained value $r_{WN} = 0.93$ indicates a strong positive association between the two variables under variable indeterminacy. This example demonstrates that the proposed weighted neutrosophic correlation coefficient effectively captures both the relationship between variables and the uncertainty inherent in the observations.

6. Discussion

The proposed model allows indeterminacy to vary across observations, which better reflects real-world data characteristics. The inclusion of weights enables prioritization of observations based on reliability or importance. These features make the proposed neutrosophic correlation coefficient more flexible and suitable for applications in decision-making, risk assessment, and uncertain data analysis.

7. Conclusion

In this paper, a weighted neutrosophic correlation coefficient with variable indeterminacy has been proposed as an extension of existing neutrosophic correlation models. The proposed coefficient preserves essential statistical properties while offering enhanced flexibility in modeling uncertainty. Future research may focus on multivariate extensions, neutrosophic partial correlation, and applications in multi-criteria decision-making problems.

8. References

[1] C. Yu, Correlation of fuzzy numbers, *Fuzzy Sets and Systems*, vol. 55, pp. 303–307, 1993.

[2] D. A. Chiang and N. P. Lin, Correlation of fuzzy sets, *Fuzzy Sets and Systems*, vol. 102, pp. 221–226, 1999.

[3] K. T. Atanassov, *Intuitionistic Fuzzy Sets*, Physica–Verlag, Heidelberg and New York, 1999.

[4] D. H. Hong and S. Y. Hwang, Correlation of intuitionistic fuzzy sets in probability spaces, *Fuzzy Sets and Systems*, vol. 75, pp. 77–81, 1995.

[5] W.-L. Hung and J. W. Wu, Correlation of intuitionistic fuzzy sets, *Fuzzy Sets and Systems*, vol. 133, pp. 69–77, 2002.

[6] F. Smarandache, *A Unifying Field in Logics: Neutrosophic Logic*, American Research Press, Philosophy, pp. 1–141, 1999.

[7] F. Smarandache, Neutrosophy: A new branch of philosophy in multiple-valued logic, *Multiple-Valued Logic: An International Journal*, vol. 8, no. 3, pp. 297–384, 2002.

[8] F. Smarandache, Neutrosophic set — a generalization of the intuitionistic fuzzy sets, in *Proceedings of the Third Conference of the European Society for Fuzzy Logic and Technology (EUSFLAT)*, Zittau, Germany, 2003, pp. 141–146.

[9] A. A. Salama and S. A. Al-Blowi, Neutrosophic set and neutrosophic topological spaces, *IOSR Journal of Mathematics*, vol. 3, no. 4, pp. 31–35, 2012.

[10] I. M. Hanafy, A. A. Salama, and K. Mahfouz, Correlation of neutrosophic data, *International Refereed Journal of Engineering and Science*, vol. 1, no. 2, pp. 39–43, 2012.

[11] I. M. Hanafy, A. A. Salama, O. M. Khaled, and K. M. Mahfouz, Correlation of neutrosophic sets in probability spaces, *Journal of Applied Mathematics, Statistics and Informatics*, vol. 10, no. 1, pp. 45–62, 2014.

[12] A. A. Salama, O. M. Khaled, and K. M. Mahfouz, Neutrosophic correlation and simple linear regression, *Neutrosophic Sets and Systems*, vol. 5, p. 3, 2014.

[13] X. Zhou, M. Lin, and W. Wang, Statistical correlation coefficients for single-valued neutrosophic sets and their applications in medical diagnosis, *AIMS Mathematics*, vol. 8, no. 7, pp. 16340–16359, 2023.

[14] S. Pramanik and K. Mondal, Cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis, *Journal of New Theory*, no. 4, pp. 90–102, 2015.