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Super Rhotrix Driven Multi-criteria Decision Making

R. Priya[1,2,3], Nivetha Martin[4], Florentine Smarandache[5]

[1]Department of Mathematics, Sethu Institute of Technology, Madurai, India
[2]Department of Mathematics, Arul Anandar College (Autonomous), Affiliated to Madurai Kamaraj University, Karumathur, India
[3]Postdoctoral Department, Turon University, Karshi 180100, Uzbekistan
[4]Post Doctoral, Department, International Engineering and Technology Institute; Hong Kong, China
[5]Department of Mathematics, University of New Mexico, USA
*Corresponding author's email: iampriyaravi@gmail.com

Abstract

This study introduces the theoretical conceptualization of Super Rhotrix and its applications in multi-criteria decision making. The properties, operations and classifications of Super Rhotrix are discoursed in this work. The geometrical interpretation of the structures of rhotrix and super rhotrix are described in the context of applications to decision-making. The rigidity and compatibility of super rhotrix is well -substantiated and demonstrated with the illustration of supplier selection problem. The merits and limitations of the proposed decision-making approach is also discussed.

Keywords: Rhotrix, Super Rhotrix, Decision making, Supplier selection

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1. Introduction

The term rhotrix is characterized as rhombus matrix with rhomboidal arrangement of elements. Ajibade [1] introduced the notion of rhotrix with the theoretical developments of rhotrix properties. Sani [2] developed an alternate method of rhotrix multiplication and introduced a generalized approach of multiplying rhotrices of higher order. This led to the evolution of rhotrix algebra. Mohammed and Sani [3] contributed to the development of rhombus graphical structures. Mohammed and Balarade [4] discussed the application of rhombus in various contexts.Sharma and Kumar [5] discoursed on Hadamard rhotrices in constructing balanced incomplete blocks. Usaini and Mohammed [6] contributed to the developments of rhotrix based eigenvalues and eigen vectors. Mohammed [7] conceptualized a new approach of expressing the notion of rhotrix. Sharma et al [8] explained the development of Hadamard codes through Hadamard codes. Kumar [9] extended polygraphicc structures suing rhotrix representations. Sharma et al [10] also focused on the structures of vandermonde rhotrix. Kaurangini and Aminu [11] explained the development of Hermitian and skew -Hermitian rhotrices.The theory of rhotrix is also applied to solve intricate problems. Anuradha et al [12] employed rhotrix approach in computing solutions to coupled linear programming problems. Isere [13] briefed on natural rhotrix and Mohammed [14] worked on non-commutative full rhotrix ring. Kavitha et al [15] formulated a prediction model based on rhotrix for weather forecasting. Verma et al [16] explained the rhotrix application in designing balanced and incomplete block designs.Ananda Priya et al [17] discoursed the classification of commutative general rhotrix group.Aminu [18] discussed fuzzy rhotrix and its applications.Ananda Priya et al [19] developed neutrosophic based rhotrix decision making model. Nivetha et al [20] discussed linguisitc rhotrix and developed decision making models based on neutrosophic linguistic rhotrices. Sathya and Nivetha [21] generalized rhotrix representations to plithogenic rhotrix.

The rhotrix structures are characterized as rhomboidal matrix. The theoretical developments of rhotrix comprising properties, operators are synonyms with matrix. The structure of matrix is extended to super matrix with each element as blocks. Researchers have applied super matrix in decision making. To mention a few, Huber et al [22] explored supermatrix analysis to determine the criterion weights in customer satisfaction. Hermann et al [23] discussed the significance of supermatrix to find the relative importance of service attributes. Cooper et al [24] discoursed disjoint supermatrix in ANP decision model. Shah et al [25] explored the applications of intuitionistic fuzzy super matrices in decision making. This has inspired and motivated the authors of this work to evolve the notion of super rhotrix. The properties and operators of rhotrix are extended to super rhotrix.

The remaining contents of the work are organized into the following section. The overview of rhotrix is presented in section 2. The theoretical developments of super rhotrix are discussed in section 3. The application of super rhotrix in decision making is demonstrated in section 4. The final section concludes the work.

2. Overview of Rhotrix

2.1 Rhotrix

A rhotrix is the rhomboidal arrangement of elements. The smallest rhotrix R of order 3 can be defined as.

$$R = \langle b \quad \begin{matrix} a \\ c \\ e \end{matrix} \quad d\rangle\text{: a,b,c,d and e} \in R$$

2.2 Heart of Rhotrix

The heart of the rhotrix is denoted by h (R) is denoted as c. It is to be mentioned that a Rhotrix is always of odd order.i.e., if there exist a Rhotrix of order n,then n must be odd. The cardinality of the Rhotrix R of order n will be $\frac{(n^2+1)}{2}$. Therefore,the cardinality of the above Rhotrix R of order 3 is 5.

2.3 Coupled matrices of a Rhotrix

The coupled matrices of Rhotrix can be obtained by rotating its columns by 45° in anti-clock ise directon instead of 90°.By doing so,first one can,we get a special matrix with missing value of order n.

$$R_5 = \left\langle \begin{matrix} & & a_{11} & & \\ & a_{21} & c_{11} & a_{12} & \\ a_{31} & c_{21} & a_{22} & c_{12} & a_{13} \\ & a_{32} & c_{22} & a_{23} & \\ & & a_{33} & & \end{matrix} \right\rangle \qquad R_5^{\frac{T}{2}} = \begin{bmatrix} a_{11} & c_{11} & a_{12} & c_{12} & a_{13} \\ a_{21} & & a_{22} & & a_{23} \\ a_{31} & c_{21} & a_{32} & c_{22} & a_{33} \end{bmatrix}$$

In this case, $\frac{T}{2}$ indicates half rotation in compare to transpose but direction issame (anti-clock ise). We observe two coupled matrices, higher order matrix A is known as major matrix and lower order matrix C is known as minor matrix,

$$A_{3\times3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \text{ and } C_{2\times2} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix}.$$

In general, for a Rhotrix of order n (Rn), one can have

$R_n^{\frac{T}{2}} = < a_{ij}, c_{ik} >^{\frac{T}{2}} = \left[A_{ij,} C_{ik}\right] = [A, C]$(say), where $i, j = 1,2,3 \ldots, t$ and $l, k = 1,2,3 \ldots (t-1)$.

So, **A** and **C** are coupled square matrices of order t and (t-1),Where $t=\frac{(n+1)}{2}$

2.4 Transpose of Rhotrix

The transpose of a Rhotrix can be obtained by exchanging row with corresponding column of a Rhotrix. Let, R be a rhotrix of order 3 as

$$R= \left\langle b \quad \begin{matrix} a \\ h(R) \\ e \end{matrix} \quad d \right\rangle \qquad \text{then } R^T = \left\langle d \quad \begin{matrix} a \\ h(R) \\ e \end{matrix} \quad b \right\rangle$$

The transpose of two Rhotrices R and Q of same order hold the property $(RQ)^T = Q^T R^T$

2.5 Addition of Rhotrix

The addition of Rhotrix is similar to matrix. For two Rhotrices of same order, add the corresponding element. Suppose, R and Q are two Rhotrices of order 3, then the addition operation can be performed as

$$R + Q = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle + \left\langle \begin{array}{ccc} & f & \\ g & h(Q) & j \\ & k & \end{array} \right\rangle$$

$$= \left\langle \begin{array}{ccc} & a+f & \\ b+g & h(R)+h(Q) & d+j \\ & e+k & \end{array} \right\rangle$$

2.6 Scalar multiplication

Multiplication of a scalar $c \in \mathbf{R}$ and a real Rhotrix R, results in multiplication of each element with the scalar . Let

$$R = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle \text{ then } cR = \left\langle \begin{array}{ccc} & aa & \\ ab & ah(R) & ad \\ & ae & \end{array} \right\rangle$$

2.7 Multiplication of Rhotrix

To establish similarity between rhotrix and matrix first we have to define row and column of rhotrix. Let, R be a Rhotrix of order 3 which is given as

$$R = \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array}$$

Then, rows of R can be viewed as (a d), (b e) whereas the columns can be viewed as (a b), (d e). Rhotrix multiplication is then same as matrix row-column multiplication applied in both the coupled matrices. For example,

$$R \circ Q = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle \circ \left\langle \begin{array}{ccc} & f & \\ g & h(R) & j \\ & k & \end{array} \right\rangle = \left\langle \begin{array}{ccc} & af+dg & \\ bf+eg & h(R)h(Q) & aj+dk \\ & bj+ek & \end{array} \right\rangle$$

2.8 Determinant of rhotrix

Let $R3 = \left\langle \begin{array}{ccc} & a & \\ b & h(R) & d \\ & e & \end{array} \right\rangle$ with the coupled matrices of R3 as $\boldsymbol{A} = \begin{bmatrix} a & d \\ b & e \end{bmatrix}$ and = $[h(R)]$. Then the

determinant of R3 is

$|R3| = |\boldsymbol{A}||\boldsymbol{C}| = h(R)(ae - bd)$

3. Conceptualization of Super Rhotrix

This section explores the theoretical development of super rhotrix. As discussed in the previous section, a rhotrix is primarily described as rhomboidal arrangement of values. The fundamental structure of rhotrix is extended to super rhotrix of order n in which each element of the super rhotrix is itself a rhotrix of same order.

3.1 Definition

A rhotrix in which each element is of a rhotrix form is called as Super Rhotrix.

$$S_R = \left\langle \begin{array}{ccc} & \langle R2\rangle & \\ \langle R1\rangle & \langle R3\rangle & \langle R5\rangle \\ & \langle R4\rangle & \end{array} \right\rangle$$

Where $\langle Ri\rangle = \left\langle a_i \begin{array}{c} b_i \\ c_i \\ d_i \end{array} e_i \right\rangle$, a_i, b_i, c_i, d_i and $e_i \in R$

The general form of the super rhotrix is

$$S_R = \left\langle \begin{array}{ccccccccc} & & & & r21 & & & & \\ & & & r22 & r23 & r24 & & & \\ & & & & r25 & & & & \\ & r11 & & & r31 & & & r51 & \\ r12 & r13 & r14 & r32 & r33 & r34 & r52 & r53 & r54 \\ & r15 & & & r35 & & & r55 & \\ & & & & r41 & & & & \\ & & & r42 & r43 & r44 & & & \\ & & & & r45 & & & & \end{array} \right\rangle$$

Example

$$S_R = \left\langle \begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 4 & 5 & 3 & & & \\ & & & & 7 & & & & \\ & 5 & & & 1 & & & 9 & \\ 0 & 8 & 3 & 4 & 3 & 6 & 3 & 2 & 7 \\ & 7 & & & 9 & & & 1 & \\ & & & & 1 & & & & \\ & & & 9 & 6 & 5 & & & \\ & & & & 3 & & & & \end{array} \right\rangle$$

3.2 Operations of Super Rhotrix

3.2.1 Transpose of Super Rhotrix

The transpose of a Super Rhotrix can be obtained by exchanging row with corresponding column of a super Rhotrix. Let, R be a super rhotrix of order 3 is

R2
$S_R =$ ⟨ R1 R3 R5 ⟩ $S_R^T =$ ⟨ R5 R3 R1 ⟩
R4   R4

(Each matrix above has R2 over the centre entry R3, and R4 under it.)

1
4 6 2
8
1 1 0
Example $S_R =$ ⟨ 0 2 3 4 2 3 5 2 3 ⟩ its transpose is
7 8 1
0
9 8 3
5

1
2 6 4
8
1 1 0
$S_R^T =$ ⟨ 3 2 0 3 2 4 3 2 5 ⟩
7 8 1
0
3 8 9
5

3.2.2 Addition of Rhotrix

The addition of two Super Rhotrix is similar to Rhotrix. For two Super Rhotrices of same order, add the corresponding element of each rhotrix. Suppose, R and S are two Super Rhotrices of order 3, then the addition operation can be performed as

R2 S2 T2
$S_R =$ ⟨ R1 R3 R5 ⟩ $S_S =$ ⟨ S1 S3 S5 ⟩ S_{R+S} = ⟨ T1 T3 T5 ⟩
R4 S4 T4

Example

Let the two Super rhotrices S_R and S_S of order 3 is

1   2
4 6 2   8 5 3
8   1
1 1 0   5 6 5
$S_R =$ ⟨ 0 2 3 4 2 3 5 2 3 ⟩ $S_S =$ 4 ⟨ 5 1 5 4 0 2 6 1 ⟩
7 8 1   3 8 4
0   3
9 8 3   1 2 5
5   7

3
12 11 5
9
6 7 5
S_{R+S} = 4 7 4 9 6 3 7 8 4
10 16 5
3
10 10 8
12

3.2.3 Scalar multiplication

Multiplication of a scalar $\alpha \in \Re$ and a real Super Rhotrix R, results in multiplication of each element with the scalar α .

$$S_R = \left\langle \begin{array}{ccc} & \mathbf{R2} & \\ \mathbf{R1} & \mathbf{R3} & \mathbf{R5} \\ & \mathbf{R4} & \end{array} \right\rangle \quad S\,\alpha\,R = \left\langle \begin{array}{ccc} & \alpha\mathbf{R2} & \\ \alpha\mathbf{R1} & \alpha\mathbf{R3} & \alpha\mathbf{R5} \\ & \alpha\mathbf{R4} & \end{array} \right\rangle$$

Example

Let us take the Super rhotrices S_R of order 3 is

$$S_R = \left\langle \begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 4 & 6 & 2 & & & \\ & & & & 8 & & & & \\ & 1 & & & 1 & & & 0 & \\ 0 & 2 & 3 & 4 & 2 & 3 & 5 & 2 & 3 \\ & 7 & & & 8 & & & 1 & \\ & & & & 0 & & & & \\ & & & 9 & 8 & 3 & & & \\ & & & & 5 & & & & \end{array} \right\rangle \quad S_{5R} = \left\langle \begin{array}{ccccccccc} & & & & 5 & & & & \\ & & & 20 & 30 & 10 & & & \\ & & & & 40 & & & & \\ & 5 & & & 5 & & & 0 & \\ 0 & 10 & 15 & 20 & 25 & 15 & 25 & 10 & 15 \\ & 35 & & & 40 & & & 5 & \\ & & & & 0 & & & & \\ & & & 45 & 40 & 15 & & & \\ & & & & 25 & & & & \end{array} \right\rangle$$

3.2.4 Multiplication of Rhotrix

Then, rows of R can be viewed as (R1 R4), (R2 R5) whereas the columns can be viewed as (R1 R2), (R4 R5). Rhotrix multiplication is then same as matrix row-column multiplication applied in both the coupled matrices.

$$S_R = \left\langle \begin{array}{ccc} & R1 & \\ R2 & R3 & R4 \\ & R5 & \end{array} \right\rangle \quad S_S = \left\langle \begin{array}{ccc} & S1 & \\ S2 & S3 & S4 \\ & S5 & \end{array} \right\rangle$$

$$S_{R^\circ S} = \left\langle \begin{array}{ccc} & R1S1 + R4S2 & \\ R2S1 + R5S2 & R3S3 & R1S4 + R4S5 \\ & R2S2 + R5S5 & \end{array} \right\rangle$$

Example

Let us consider the two rhotrix S_R and S_S is

$$S_R = \left\langle \begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 4 & 6 & 2 & & & \\ & & & & 8 & & & & \\ & 1 & & & 1 & & & 0 & \\ 0 & 2 & 3 & 4 & 2 & 3 & 5 & 2 & 3 \\ & 7 & & & 8 & & & 1 & \\ & & & & 0 & & & & \\ & & & 9 & 8 & 3 & & & \\ & & & & 5 & & & & \end{array} \right\rangle \quad S_S = \left\langle \begin{array}{ccccccccc} & & & & 2 & & & & \\ & & & 8 & 5 & 3 & & & \\ & & & & 1 & & & & \\ & 5 & & & 1 & & & 5 & \\ 4 & 5 & 1 & 4 & 2 & 3 & 2 & 6 & 1 \\ & 3 & & & 8 & & & 4 & \\ & & & & 3 & & & & \\ & & & 1 & 2 & 5 & & & \\ & & & & 7 & & & & \end{array} \right\rangle$$

```
                 18
             72  30   5
                 20
     17          13           6
S_R∘S = 28   10   10   36   4   27   27   12   12
     21          76           9
                  3
             32  16  21
                 80
```

3.2.5 Determinant of rhotrix

```
            R1
Let S_R =   R2   R3   R4  〉
            R5
```

with the coupled matrices of S_R as $\boldsymbol{A}=\begin{bmatrix} R1 & R4 \\ R2 & R5 \end{bmatrix}$ and $\boldsymbol{C}$=[R3].

$|S_R| = |\boldsymbol{A}||\boldsymbol{C}| = R3(R1R5-R4R2)$

Example

Consider the Super rhotrix

```
                    1
                 4  6  2
                    8
          1         1         0
S_R = 〈 0  2   3   4   2   3   5   2   3 〉 then |S_R| =   6  〈 −190
          7         8         1                                  58  −10 〉
                    0                                          −360
                 9  8  3
                    5
```

3.3 Properties of Super Rhotrix

```
                                                                      0
<R +> is a commutative group same as matrix, with identity element O = 0  0  0 〉
                                                                      0
```

For addition of two Super Rhotrices, following are satisfied.

- **Closed under operation**: If S_R and S_Q are two real Super Rhotrices of same order n then their addition is also a real Super Rhotrix of same order.

- **Associative**: If there exist another Super Rhotrix S_P of same order n, then $S_R + (S_Q + S_P) = (S_R + S_Q) + S_P$.

- **Additive Inverse**: $S_R + S_Q = O$, where O is identity element then $S_Q = -S_R$.

- **Multiplicative Inverse :** A multiplicative inverse or reciprocal for a rhotrix S_R, denoted by R^{-1}, is a super rhotrix which when multiplied by S_R yields the multiplicative identity i.e., an identity super rhotrix I.

- **Multiplicative identity :** The identity property of multiplication states that when any Super Rhotrix I is multiplied by any other rhotrix S_R, the Super rhotrix S_R does not change. That is $S_R \circ S_I = S_I \circ S_R = S_R$

3.4 Classification of Super Rhotrix

The super rhotrix shall be primarily classified as Quantitative and Qualitative.

3.4.1 Quantitative Super Rhotrix

A super rhotrix with numerical values is called as Quantitative Super Rhotrix

Example

$$S_R = \left\langle \begin{array}{ccccccccc} & & & & 3 & & & & \\ & & & 5 & 2 & 1 & & & \\ & & & & 8 & & & & \\ & 4 & & & 9 & & & 3 & \\ 5 & 3 & 7 & 6 & 4 & 5 & 8 & 2 & 7 \\ & 8 & & & 3 & & & 6 & \\ & & & & 6 & & & & \\ & & & 9 & 2 & 1 & & & \\ & & & & 7 & & & & \end{array} \right\rangle$$

3.4.2 Qualitative Super Rhotrix

A super rhotrix with linguistic variables is called as Qualitative Super Rhotrix

Example

$$S_R = \left\langle \begin{array}{ccccccccc} & & & & L & & & & \\ & & & VL & H & H & & & \\ & & & & M & & & & \\ & M & & & H & & & L & \\ H & VH & L & VL & M & L & H & VH & M \\ & H & & & VH & & & M & \\ & & & & H & & & & \\ & & & VH & M & VL & & & \\ & & & & L & & & & \end{array} \right\rangle$$

3.4.3 Classification of Qualitative Super Rhotrix

3.4. 4 Fuzzy Super Rhotrix

A super rhotrix is said to be fuzzy super rhotrix if the linguistic variables are quantified using fuzzy representations.

Example

$$\left\langle \qquad \right\rangle$$

$$
\begin{array}{ccccccccc}
 & & & & 0.4 & & & & \\
 & & & 0.2 & 0.8 & 0.8 & & & \\
 & & & & 0.6 & & & & \\
 & 0.6 & & & 0.8 & & & 0.4 & \\
0.8 & 1 & 0.4 & 0.2 & 0.6 & 0.4 & 0.8 & 1 & 0.6 \\
 & 0.8 & & & 1 & & & 0.6 & \\
 & & & & 0.8 & & & & \\
 & & & 1 & 0.6 & 0.2 & & & \\
 & & & & 0.4 & & & &
\end{array}
$$

3.4. 5 Intuitionistic Fuzzy Super Rhotrix

A super rhotrix is said to be intuitionistic fuzzy super rhotrix if the linguistic variables are quantified using intuitionistic fuzzy representations.

Example

$$
S_R = \left\langle
\begin{array}{ccccccccc}
 & & & & (0.3,0.7) & & & & \\
 & & & (0.1,0.9) & (0.8,0.2) & (0.8,0.2) & & & \\
 & & & & (0.5,0.5) & & & & \\
 & (0.5,0.5) & & & (0.8,0.2) & & & (0.3,0.7) & \\
(0.8,0.2) & (0.9,0.1) & (0.3,0.7) & (0.1,0.9) & (0.5,0.5) & (0.3,0.7) & (0.8,0.2) & (0.9,0.1) & (0.5,0.5) \\
 & (0.8,0.2) & & & (0.9,0.1) & & & (0.5,0.5) & \\
 & & & & (0.8,0.2) & & & & \\
 & & & (0.9,0.1) & (0.5,0.5) & (0.1,0.9) & & & \\
 & & & & (0.3,0.7) & & & &
\end{array}
\right\rangle
$$

3.4. 6 Neutrosophic Super Rhotrix

A super rhotrix is said to be neutrosophic super rhotrix if the linguistic variables are quantified using neutrosophic representations.

Example

$$
S_R = \left\langle
\begin{array}{ccccccc}
 & & & & (0.5,0.95,0.95) & & \\
 & & & (0.25,0.75,0.75) & (0.75,0.25,0.25) & (0.75,0.25,0.25) & \\
 & & & & (0.5,0.5,0.5) & & \\
 & (0.5,0.5,0.5) & & & (0.75,0.25,0.25) & & \\
(0.75,0.25,0.25) & (0.9,0.5,0.5) & (0.5,0.95,0.95) & (0.25,0.75,0.75) & (0.5,0.5,0.5) & (0.5,0.95,0.95) & (\\
 & (0.75,0.25,0.25) & & & (0.9,0.5,0.5) & & \\
 & & & & (0.75,0.25,0.25) & & \\
 & & & (0.9,0.5,0.5) & (0.5,0.5,0.5) & (0.25,0.75,0.75) & \\
 & & & & (0.5,0.95,0.95) & &
\end{array}
\right\rangle
$$

4. Illustration

This section presents the application of super rhotrix in supplier selection oriented decision making. In general, the supplier selection problem is well handled using the general procedure of multi-criteria decision making. The alternatives and the criteria are initially decided and then the decision matrix is formulated. However, to understand rhotrix driven decision-making, let us first explore the geometrical structure of rhotrix and super rhotrix.

4.1 Geometrical Interpretation of Rhotrix in Decision Making

The rhomboidal arrangement of the rhotrix is not arbitrary, the representation with 5-position shall be interpretated as follows in the context of depicting a decision making problem. The position and description of the elements is presented in Table 1.

The general rhotrix is of the form $\left\langle \begin{array}{ccc} & b & \\ a & c & e \\ & d & \end{array} \right\rangle$

Table 1 Description of Rhotrix.

Symbol	**Position**	**Description**
A	centre-left	Control / influence input 1
B	top-middle	Secondary indicator 1
C	centre-middle	**Main indicator (core value)**
D	bottom-middle	Secondary indicator 2
E	centre-right	Control / influence input 2

Let us map the rhotrix structure to the representation of supplier selection criteria presented in Table 2.

Table 2 Criteria representation in Rhotrix structure

Criteria of Supplier Selection	**Symbol**	**Description**
Price	A	primary control variable
Quality	B	performance-oriented indicator
Sustainability	C	core evaluation criterion
Delivery	D	supporting operational metric
Service	E	controllable managerial input

4.2 Conceptual Rhotrix Representation

$$\left\langle \begin{array}{ccc} & \text{Quality (b)} & \\ \text{Price (a)} & \text{Sustainability (c)} & \text{Service (e)} \\ & \text{Delivery (d)} & \end{array} \right\rangle$$

This conceptual rhotrix structure facilitates both hierarchical and interactive representation of supplier selection criteria, with controllable inputs as price and service, indicators as quality and delivery and with the core criterion value of sustainability.

Let us employ this rhotrix representation to represent the supplier performance subjected to the above-mentioned criteria.

$$S = \langle 0.75,\ 0.65,\ 0.55,\ 0.50,\ 0.60 \rangle \Rightarrow \left\langle \begin{array}{ccc} & 0.65 & \\ 0.75 & 0.55 & 0.60 \\ & 0.50 & \end{array} \right\rangle$$

Let us assume the criterion weights say $w_k = (w_a, w_b, w_c, w_d, w_e)$

In this case, $w_k = (0.30, 0.25, 0.20, 0.15, 0.10)$ such that $\sum_{k=1}^{m} w_k = 1$

The score of the supplier is computed as

$$\text{score}(r) = aw_a + bw_b + cw_c + dw_d + ew_e$$

$$= 0.6325$$

This gives the score of the supplier. This rhotrix representation has given the score of the supplier.

4.3 Extension of Rhotrix to Super Rhotrix

Now let us extend and apply the super rhotrix representation to rank the suppliers of a company based on each criterion.

$$\left\langle \begin{array}{ccc} & \langle S2 \rangle & \\ \langle S1 \rangle & \langle S3 \rangle & \langle S5 \rangle \\ & \langle S4 \rangle & \end{array} \right\rangle$$

In the above representation, each element is a rhotrix representing the supplier information. The positioning of the suppliers do not follow any specific order.

$$
\left\langle
\begin{array}{ccccccccc}
 & & & & 0.75 & & & & \\
 & & & 0.85 & 0.65 & 0.70 & & & \\
 & & & & 0.60 & & & & \\
 & 0.85 & & & 0.60 & & & 0.60 & \\
0.70 & 0.80 & 0.65 & 0.90 & 0.55 & 0.60 & 0.90 & 0.55 & 0.60 \\
 & 0.50 & & & 0.80 & & & 0.80 & \\
 & & & & 0.80 & & & & \\
 & & & 0.60 & 0.75 & 0.85 & & & \\
 & & & & 0.55 & & & &
\end{array}
\right\rangle
$$

The weighted scores of each supplier subjected to each of the criteria is presented in Table 3.

Table 3. Scores of the Alternatives

	S1	S2	S3	S4	S5
Price	0.21	0.255	0.27	0.18	0.27
Quality	0.2125	0.1875	0.15	0.2	0.15
Sustainability	0.16	0.13	0.11	0.15	0.11
Delivery	0.075	0.09	0.12	0.0825	0.12
Service	0.065	0.07	0.06	0.085	0.06

The aggregate scores and rankings of the suppliers is presented in Table 4.

Table 4 Ranking of the Alternatives

	Aggregate Scores	Rank
S1	0.7225	2
S2	0.7325	1
S3	0.71	3
S4	0.6975	5
S5	0.71	3

The rankings obtained using super rhotrix driven approach is simple and viable. The criterion representation is more realistic and interactive. The proposed super rhotrix based results shall be validated with other multi-criteria decision making methods. In comparison with the genre of MCDM approaches, the proposed super rhotrix based decision making is feasible with time and cost efficacy. However, the super rhotrix representation may not be compatible with increasing number of alternatives and criteria. This is identified as one of the limitations of the proposed decision approach.

5. Conclusion

This study introduces the notion of super rhotrix and discourses the applications of super rhotrix driven decision making approach. The proposed architecture of super rhotrix is well described and substantiated with suitable illustrations. This extended rhotrix shall be further discoursed and discussed in other domains of decision making. The super rhotrix structure shall be further modified to overcome the limitation of criteria and alternative representations.

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